DISPERSION ANALYSIS FOR GENERALIZED SPIN POLARIZABILITIES

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We report on a dispersion relation formalism for the virtual Compton scattering (VCS) reaction on the proton, which for the first time allows a dispersive evaluation of 4 generalized polarizabilities. The dispersion formalism provides a new tool to analyze VCS experiments above pion threshold, thus increasing the sensitivity to the generalized polarizabilities of the nucleon.

1 Introduction

Over the past years, the virtual Compton scattering (VCS) process on the proton, accessed through the $ep \to ep\gamma$ reaction, has become a powerful tool to provide new information on the internal structure of the nucleon [1,2].

In the low energy regime below pion threshold, the outgoing photon in the VCS process plays the role of a quasi-constant applied electromagnetic dipole field and, through electron scattering, one measures the spatial distribution of the nucleon response to this applied field. The response is parametrized in terms of 6 generalized polarizabilities (GP's) [3,4], which are functions of the square of the virtual photon four-momentum Q^2. The GP's provide valuable non-perturbative nucleon structure information, and have been calculated in different approaches. In particular, the GP's teach us about the interplay between nucleon-core excitations and pion-cloud effects.

The first dedicated VCS experiment has been performed at MAMI [5] and two combinations of GP's have been determined at $Q^2 = 0.33$ GeV^2. Further VCS experiments are underway at lower Q^2 at MIT-Bates [6] and at higher Q^2 at JLab [7].

At present, VCS experiments at low outgoing photon energies are analyzed in terms of a low-energy expansion (LEX) as proposed by Guichon et al. [3], assuming that the non-Born (i.e. nucleon excitation) response to the quasi-constant electromagnetic field is exclusively given by its leading term in the outgoing photon energy q′. This term depends linearly on the GP's. As the sensitivity of the VCS cross sections to the GP's grows with the photon energy, it is advantageous to go to higher photon energies, provided one can keep the theoretical uncertainties under control when crossing the pion threshold. The situation can be compared to real Compton scattering (RCS), for which one uses a dispersion relation formalism [8,9] to extract the polarizabilities at energies above pion threshold, with generally larger effects on the observables. We report here on recent work [10] to set up a dispersion formalism for VCS. This provides a tool to analyze VCS experiments at higher energies in order to extract the GP's from data over a larger energy range. It will be shown that the same formalism also provides for the first time a dispersive evaluation of 4 GP's.

2 Dispersion formalism for VCS

The nucleon structure information obtained through VCS can be parametrized in terms of 12 non-Born invariant amplitudes, denoted by $F_i^{NB} (i = 1, ..., 12)$ [10]. The F_i^{NB} are functions of 3 invariants for the VCS process : Q^2, $\nu = (s-u)/(4M_N)$, and t (s, t and u are the Mandelstam invariants for VCS, and M_N denotes the nucleon mass).

Assuming analyticity and an appropriate high-energy behavior, the non-Born amplitudes $F_i^{NB}(Q^2, \nu, t)$ fulfill unsubtracted dispersion relations (DR's) with respect to the variable ν at fixed t and fixed virtuality Q^2 :

$$\mathrm{Re}F_i^{NB}(Q^2, \nu, t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{thr}}^{+\infty} d\nu' \frac{\nu' \, \mathrm{Im}_s F_i(Q^2, \nu', t)}{\nu'^2 - \nu^2}, \qquad (1)$$

with $\mathrm{Im}_s F_i$ the discontinuities across the s-channel cuts of the VCS process.

It has been shown [10] that 10 of the 12 invariant amplitudes drop sufficiently fast at high energy and can be evaluated through unsubtracted dispersion integrals as in Eq. (1). The remaining two amplitudes, denoted by F_1 and F_5, cannot be evaluated through an unsubtracted dispersion integral. This situation is similar to RCS, where 2 of the 6 invariant amplitudes cannot be evaluated by unsubtracted dispersion relations either [8].

The imaginary parts $\mathrm{Im}_s F_i$ in Eq. (1) are calculated by use of unitarity. In our calculation, we saturate the dispersion integrals by the dominant contribution of the πN intermediate states. For the pion photo- and electroproduction helicity amplitudes, we use the MAID analysis [11], which contains both resonant and non-resonant pion production mechanisms.

3 Dispersion results for generalized polarizabilities

To obtain dispersive estimates for the GP's, one first expresses the GP's in terms of the VCS amplitudes F_i^{NB} at the point $\nu = 0$, $t = -Q^2$ at finite Q^2, for which we introduce the shorthand : $\bar{F}_i(Q^2) \equiv F_i^{NB}\left(Q^2, \nu = 0, t = -Q^2\right)$. The relations between the GP's and the $\bar{F}_i(Q^2)$ can be found in Ref. [4].

Unsubtracted DR's for the GP's hold for those combinations of GP's that do not depend upon the amplitudes $\bar{F}_1$ and $\bar{F}_5$. Four such combinations of GP's have been found [10] and are shown in Fig. 1 : one combination contains the scalar GP's, whereas the three other combinations involve the spin GP's (see Ref. [1] for the notations of the GP's).

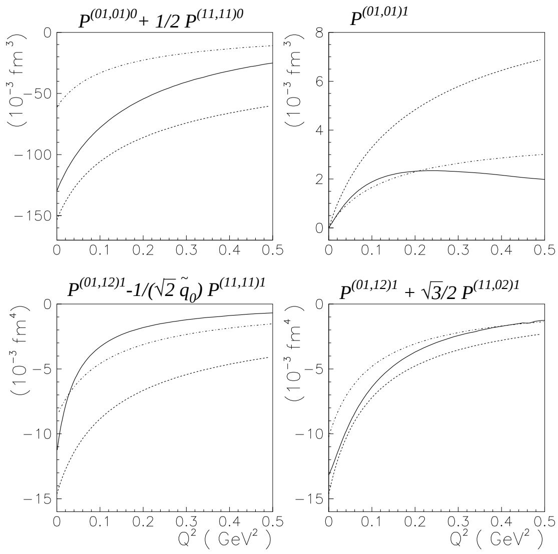

Figure 1. Dispersion results for 4 of the GP's of the proton (full curves), compared with results of $O(p^3)$ HBChPT [12] (dashed curves) and the linear σ-model [13] (dashed-dotted curves).

The dispersion results are compared in Fig. 1 with the results of the $O(p^3)$ heavy-baryon chiral perturbation theory (HBChPT) [12] and the linear σ-model [13]. A comparison with those of HBChPT at $O(p^3)$ shows that a rather good agreement for $P^{(01,12)1} + \sqrt{3}/2 P^{(11,02)1}$ is obtained, whereas for the GP's $P^{(01,01)1}$ and $P^{(01,12)1}$ - $1/(\sqrt{2}\,\tilde{q}_0) P^{(11,11)1}$, the dispersive results drop much faster with Q^2. This trend is also seen in the relativistic linear σ-model, which takes account of some higher orders in the chiral expansion.

4 Dispersion results for VCS observables

To construct the remaining two VCS amplitudes (F_1 and F_5) in an unsubtracted dispersion framework, one can proceed in an analogous way as has been proposed by L'vov [8] in the case of RCS. The unsubtracted dispersion integrals are firstly evaluated along the real ν-axis in a finite range $-\nu_{max} \leq \nu \leq +\nu_{max}$ (with $\nu_{max} \approx 1.5$ GeV). The remaining asymptotic contributions (F_1^{as} and F_5^{as}) are then approximately parametrized by t-channel poles. As a first step, we have parametrized F_1^{as} through a σ-pole and F_5^{as} through a π^0-pole, in analogy with the RCS case.

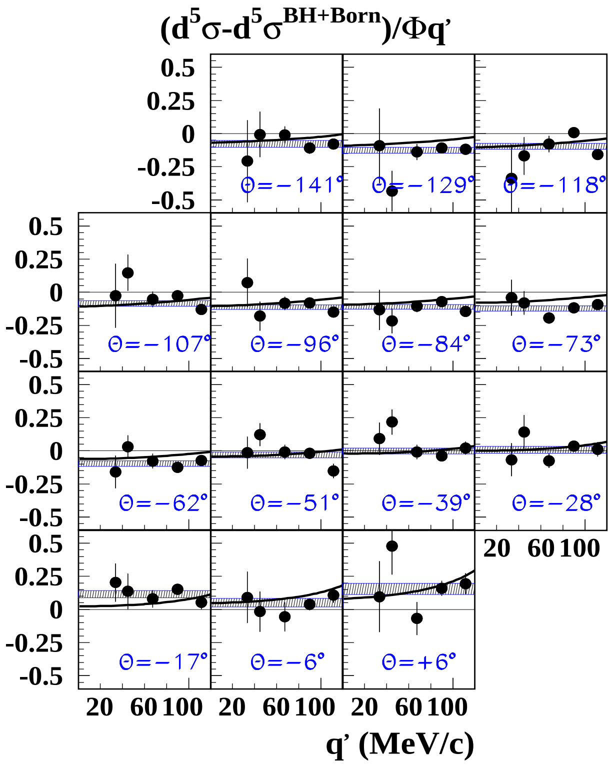

Figure 2. Photon energy dependence of the difference between the total VCS and Bethe-Heitler (BH) + Born cross sections, for different photon angles in the kinematics of the MAMI VCS experiment [5] : $Q^2 = 0.33$ GeV^2, $\varepsilon = 0.62$. The LEX gives a constant energy dependence, and is shown by the band, which represents a fit to the data [5]. The dispersion calculation is represented by the solid curves.

The resulting dispersive estimates are shown in Fig. 2 and compared with the MAMI VCS data below pion threshold. It is seen that in the low energy region (below pion threshold), the dispersive results seem to support the LEX analysis, represented by the band in Fig. 2, from which two combinations of GP's have been extracted in Ref. [5]. It remains to be seen how more refined parametrizations for the asymptotic contributions as well as an estimate of $\pi\pi N$ intermediate states in the dispersion integrals affect this result.

The next step is then to apply the present VCS dispersion formalism to higher energies in order to extract the nucleon GP's over a larger range of energies from both unpolarized and polarized VCS data.

Acknowledgments

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